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Piezoelectric Current from Shock-Loaded Quartz—A Submicrosecond Stress Gauge

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Current from X-cut quartz disks may be used to detect stress–time profiles induced by shock loading. The current amplitude and its time dependence are functions of the dielectric, piezoelectric, and mechanical properties of quartz under shock-loading conditions. The results of an extensive experimental study of the current from shock-loaded quartz disks are reported for shock stress up to 50 kbar. The experiment is performed by impacting precisely aligned X-cut quartz disks upon each other at various measured velocities and observing the current in one of the disks during the first wave transit. Within the low signal range, the piezoelectric stress constant e_{11} is found to be 0.174 C-m^{-2}. The coefficient relating current jump to stress jump in one-dimensional strain is found to be 2.04×10^{-8} C-cm^{-2}-kbar^{-1} up to 6 kbar and 2.15×10^{-8} C-cm^{-2}-kbar^{-1} from 9 to 18 kbar. The wave velocity was determined to be constant to 25 kbar. The observed current waveform could be fully interpreted in terms of rate-independent properties. Determinations of distortions to the current from apparently minor deviations from one-dimensional conditions were also made.

INTRODUCTION

Several recent articles[1–3] have demonstrated that current from X-cut quartz disks may be used for continuous observation on a submicrosecond time scale of stress–time profiles produced by shock loading. This quartz gauge is useful for stress up to 10's of kilobars with time resolution limited only by the planarity of the wave and the frequency response of the system used to record the gauge signal. The superior time resolution and sensitivity of the quartz gauge have enabled observations of mechanical behavior of solids under shock loading that are not possible with slower time-resolution techniques. A typical stress–time profile such as may be recorded on a routine basis is shown in Fig. 1.

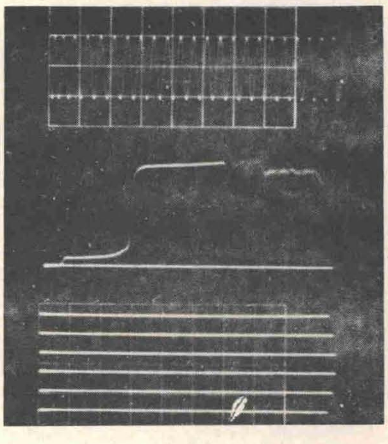

Fig. 1. Typical stress–time record obtained by recording the current from an X-cut quartz disk in direct contact with a shock-loaded specimen of a nickel–iron alloy. Time increases from left to right. 10-Mc/sec timing wave. Stress amplitude for the second wave is 45 kbar in the alloy sample.

The use of a quartz disk as a gauge requires the verification of basic assumptions concerning the physical properties of quartz, as well as quantitative clarification of the effects of minor deviations from the idealized conditions. It is the object of this paper to describe in quantitative detail the current–time characteristics of shock-loaded quartz disks in various configurations. Considerable error in interpretation of the current–time history is possible if the current distortions as outlined here are not considered.

PIEZOELECTRIC CURRENT ANALYSIS

The superior time resolution of the quartz gauge is a consequence of the observation of the short-circuit piezoelectric current during the time that the stress wave propagates along the thickness of the gauge. This is in contrast to the more conventional technique[4,5] of observing the piezoelectric charge on a time scale which is long compared to stress wave transit time in the gauge. In the latter case, the entire disk is assumed to be uniformly stressed at a given instant of time in a quasistatic state, and the piezoelectric charge is proportional to the stress in the disk. For the gauge considered in the present paper the piezoelectric current is proportional to the difference in stress at the electrodes of the disk. To understand the operation of the gauge, we must consider the piezoelectric current produced during the propagation of a stress wave along the x axis of a quartz disk.

Consider an X-cut quartz disk to one face of which a rapidly changing impulsive load is applied. Assume: (a) that the stressed region of the disk is in a state of one-dimensional strain, (b) that the electric fields produced by the piezoelectric effect are one-dimensional, (c) that the stress is instantaneously applied to the

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entire electrode face of the disk, (d) that an electric short-circuit exists between the two faces of the disk, and (e) that the strain is infinitesimal.

Assume the following about the physical properties of quartz: (f) that all stress amplitudes are steady and travel with the same wave velocity, (g) that the wave velocity is steady, (h) that the conductivity is zero, (i) that the dielectric permittivity does not change with finite shock stress and finite electric field, and (j) that the piezoelectric polarization is directly proportional to the x component of the stress by a coefficient f that is independent of time and stress for a given stress range.

Restricting the analysis to the one-dimensional case [assumptions (a) and (b)], the displacement current i due to the stress wave is

$$i=A\,dD/dt, \tag{1}$$

where A is the electroded area, D is the electric displacement (defined as $D=P+\epsilon E$), where P is the piezoelectric polarization, ϵ is the dielectric permittivity, and E is the electric field. We write the electric displacement as

$$\int_0^l D(x)dx=\int_0^l P(x)dx+\int_0^l \epsilon E(x)dx, \tag{2}$$

where l is the thickness of the disk, and the limits of integration chosen imply infinitesimal strain [assumption (e)]. Because of the short circuit [assumption (d)] and the constant permittivity [assumption (i)],

$$\int_0^l \epsilon E(x)dx=0. \tag{3}$$

For zero conductivity [assumption (h)], $\partial D/\partial x=0$ so that Eq. (2) now reduces to

$$D=\frac{1}{l}\int_0^l P(x)dx. \tag{4}$$

Further, $P(x)=f\sigma(x)$ [assumption (j)] and since the quartz is linearly elastic [assumptions (f) and (g)], $\sigma(x,t)=\sigma(x-U_st)$, where U_s is the wave propagation velocity. The solution for the current from Eqs. (1) and (4) is then

$$i=A\frac{dD}{dt}=-\frac{fAU_s}{l}\int_0^l \frac{\partial\sigma(x)}{\partial x}dx=\frac{fAU_s}{l}[\sigma_0-\sigma_l], \tag{5}$$

where σ_0 is the x component of stress at the stress input electrode and σ_l is the x component of stress at the rear electrode. For times less than wave transit time

$$\sigma_0=(l/fAU_s)i,\quad 0<t<l/U_s. \tag{6}$$

Equation (6) predicts that the stress at the input electrode is directly proportional to the instantaneous current for times less than wave transit time. For times greater than wave transit time the stress difference between the two electrodes is directly proportional to the current.

EXPERIMENTAL TECHNIQUE

In order to study the physical properties of X-cut quartz and thereby determine the validity of assumptions (f)–(j), and to determine the behavior of various quartz disks insofar as assumptions (a)–(d) are concerned, an extensive experimental program has been accomplished. The experimental technique was designed to study the instantaneous relationship between current and stress. A number of improvements have been made since the technique was first reported.[6,7]

A well-defined shock stress is imparted to a specimen disk of X-cut quartz by the impact of a precisely aligned X-cut quartz disk used as a projectile facing, whose velocity at impact is accurately measured. This experimental configuration is shown schematically in Fig. 2. X-cut quartz exhibits a single wave structure for stress levels up to about 50 kbar[8,9]; consequently, we may compute the input stress to the specimen disk in the single wave region from the conservation of momentum relation

$$\sigma=\rho_0U_su_p, \tag{7}$$

where σ is the x component of stress imparted by the wave front, ρ_0 the density of the material ahead of the wave front, U_s the wave front propagation velocity, and u_p is the particle velocity imparted by the wave front. Because the impacting and impacted materials are the same, $u_p=\frac{1}{2}U_0$, where U_0 is the measured impact velocity.

If all assumptions are met unequivocally, Eq. (6) predicts the current, due to the constant particle velocity, to be constant for full wave transit time. The experimental method was to impart an accurately

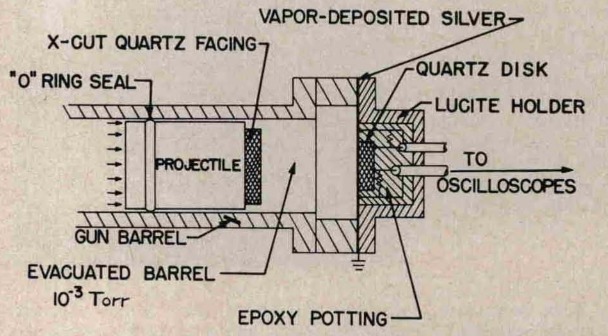

FIG. 2. Schematic of the experimental arrangement used to study the current from shock-loaded quartz disks.

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